RB-0764

Second Year B. Sc. (I.C.) Examination

April / May – 2010

Mathematics : Paper - III

Time : 3 Hours] [Total Marks : 105

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ S.Y. B.Sc. (I.C.)

Name of the Subject :
☛ MATHEMATICS - 3

☛ Subject Code No. : 0 7 6 4 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All the questions are compulsory.

(3) Digits to the right indicate marks of question.

(4) Follow usual notations.

1 Answer the following in short : **15**

(1) If $u = \tan^{-1}\left(\frac{x}{y}\right); y \neq 0$ then find $xu_x + yu_y$

(2) If $x = r\cos\theta,\ \ y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$

(3) Define maximum value of a bivariate function

(4) Solve $\left(D^2 + 8D + 25\right)y = 0$

(5) Eliminate a and b from $z = (x + a)(y + b)$

2 (a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then prove that, **6**

(i) $u_x + u_y + y_z = \frac{3}{x + y + z}$

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}$

(b) Discuss the continuity of the function, **6**

$$f(x, y) = \frac{x^2 + y^2}{x + y}; \qquad x + y \neq 0$$
$$= 1 \qquad ; \qquad x + y = 0$$

at the point $(1,1)$.

(c) Prove that, **6**

$$e^{ax+by} = 1 + (ax+by) + \frac{(ax+by)^2}{2!} + \frac{(ax+by)^3}{3!} + \ldots\ldots\ldots.$$

OR

2 (a) If $z(x+y) = x^2 + y^2$ then prove that, **6**

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

(b) Prove that f is a differentiable homogeneous function in x, y of degree m if and only if $xf_x + yf_y = mf(x, y)$. **6**

(c) If $x + y + z = u$, $y + z = uv$, $z = uvw$ then obtain $J(x, y, z)$ with respect to u, v, w. **6**

3 (a) Find the extreme values of, **9**

$$f(x, y) = x^3 + y^3 - 3x - 12y + 5$$

(b) After changing the order of integraton evaluate, **9**

$$\int_0^1 \int_y^{2-y} \left(x^2 + y^2\right) dx dy$$

OR

3 (a) State and prove the necessary conditions for function $f(x, y)$ to have extreme values at point (a, b). **9**

(b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using double integrals. **9**

4 (a) In usual notations prove that, **8**

$$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}; \; f(a) \neq 0$$

(b) Solve : **10**

(1) $\left(x^2 D^2 + 2xD - 20\right) y = (x+1)^2$

(2) $\left(D^3 - 1\right) y = e^x \cos x$

OR

4 (a) In usual notations prove that, **8**

$$\frac{1}{f(D)}[x \cdot V] = \left[x - \frac{1}{f(D)} \cdot f'(D)\right] \frac{1}{f(D)} \cdot V,$$

where V is a function of x.

(b) Solve : **10**

(1) $\left(D^4 + 2D^2 + 1\right) y = e^x + \sin x$

(2) $\left(x^2 D^2 - 3xD + 4\right) y = x^2 \log x$.

5 (a) In usual notations prove that, **8**

$\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \cdot \frac{1}{f(D+a)} \cdot V$; where V is a function of x.

(b) Solve :

(1) $\left(D^2 - 7D + 10\right) y = e^{5x} + e^{2x}$

(2) $\left(D^2 + 4\right) y = \sin 4x + x$.

OR

5 (a) Explain the method of solving the equation of type $F(z, p, q) = 0$ **8**

(b) Solve : **10**

(1) $(y-x)(qy-px) = (p-q)^2$

(2) $p^2 = z^2(1-pq)$

6 (a) Explain how to find complete and general integral of partial differential eqution, $f_1(x, p) = f_2(y, q)$ **8**

(b) Solve : **10**

(1) $x^2p^2 + y^2q^2 = z^2$

(2) $z = px + qy - 2\sqrt{pq}$

OR

6 (a) Explain the method of solving the equation of type $F(p, q) = 0$ **8**

(b) Solve : **10**

(1) $(mz-ny)p + (nx-lz)q = ly - mx$

(2) $\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$
